

Meta-Stable Brane Configurations with Five NS5-Branes

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Abstract

From an $\mathcal{N} = 1$ supersymmetric electric gauge theory with the gauge group $SU(N_c) \times SU(N'_c)$ with fundamentals for each gauge group, the bifundamentals and a symmetric flavor and a conjugate symmetric flavor for $SU(N_c)$, we apply Seiberg dual to each gauge group independently and obtain two $\mathcal{N} = 1$ supersymmetric dual magnetic gauge theories with dual matters including the gauge singlets. By analyzing the F-term equations of the dual magnetic superpotentials, we describe the intersecting brane configurations of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of these gauge theories. The case where the above symmetric flavor is replaced by an antisymmetric flavor is also discussed.

1 Introduction

Starting from $\mathcal{N} = 1$ supersymmetric gauge theory with massive fundamentals, the construction of meta-stable supersymmetry breaking vacua was found in [1] by using the corresponding dual magnetic gauge theory. The magnetic theory does have superpotential consisting of an interaction between the meson and dual quarks as well as a linear term in the meson that can be interpreted as a mass term for the quarks in the electric theory. By rank condition, the F-term equation from the dual magnetic superpotential cannot be satisfied and the supersymmetry is broken. See the review paper [2] for the recent developments of dynamical supersymmetry breaking.

In the type IIA brane configuration [3], the above gauge theory can be described by two NS5-branes, D6-branes and D4-branes. By standard brane motion, the magnetic brane configuration or Seiberg dual can be constructed from electric one. The deformation of quark mass corresponds to the relative displacement of D6-branes and D4-branes along the common orthogonal directions. The geometric misalignment of flavor D4-branes in the magnetic brane configuration can be interpreted as a nontrivial F-term equation we mentioned in the magnetic gauge theory.

When an adjoint matter is included, then one should consider a set of coincident NS5-branes rather than a single NS5-brane. When we add an orientifold 4-plane(O4-plane) to the above brane configuration, then the gauge group will be changed into a symplectic or orthogonal gauge groups. On the other hand, if we include an orientifold 6-plane(O6-plane), then the matter contents will be different due to the projection. Totally, the three NS5-branes are present and a middle NS5-brane is located at an orientifold 6-plane for the unitary gauge group [3]. All of these considerations have a single gauge group with corresponding matters.

What happens when we consider product of two gauge groups? Without any orientifold plane, there exist three NS5-branes, D6-branes and D4-branes. When we add an orientifold 4-plane to this brane configuration, then the gauge group will be changed into product gauge group of a symplectic and orthogonal gauge groups. On the other hand, if we include an orientifold 6-plane, then the matter contents will be different, in general. When there is no NS5-brane on an orientifold 6-plane, four NS5-branes are present and the gauge group will be a product of unitary gauge group and orthogonal or symplectic gauge group. On the other hand, if the NS5-brane is located at an orientifold 6-plane, then five NS5-branes are needed and the gauge group will be a product of unitary gauge groups.

In this paper, we consider a particular product gauge group $SU(N_c) \times SU(N'_c)$ with fundamentals for each gauge group, the bifundamentals as well as a symmetric and conjugate

symmetric flavor for $SU(N_c)$. Without these symmetric and conjugate symmetric flavors, the type IIA brane configuration for this product gauge group with fundamentals and bifundamentals consists of three NS5-branes, D6-branes and D4-branes for each gauge group [4, 3]. For purely gauge theory analysis, see [5, 6] for details. On the other hand, if we ignore the second gauge group $SU(N'_c)$ with corresponding matter contents completely, this theory will reduce to a single gauge group $SU(N_c)$ with a symmetric flavor, conjugate symmetric flavor and fundamental flavors developed in [6, 7]: there are three NS5-branes, D6-branes, D4-branes and an orientifold 6-plane where a middle NS5-brane is located. In addition to these branes, we add the extra two outer NS5-branes in a \mathbf{Z}_2 symmetric way due to the O6-plane, extra D4-branes and extra D6-branes corresponding to the second gauge group $SU(N'_c)$. Starting from this $\mathcal{N} = 1$ supersymmetric gauge theory with massive fundamentals for the each gauge group, we will analyze the meta-stable supersymmetry breaking brane configuration.

The product gauge group $SU(N_c) \times SU(N'_c)$ with fundamentals for each gauge group, the bifundamentals as well as an antisymmetric and conjugate symmetric flavor for $SU(N_c)$ is also considered.

In section 2, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with matter contents and deform this theory by adding the mass term for the quarks for each gauge group. Then we construct the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory with corresponding dual matters as well as various gauge singlets, by brane motion and linking number counting. Similarly, we construct the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(N_c) \times SU(\tilde{N}'_c)$ gauge theory with corresponding dual matters as well as various gauge singlets.

In section 3, we consider the nonsupersymmetric meta-stable minimum by looking at the magnetic brane configurations we obtained in section 2, present the corresponding intersecting brane configurations of type IIA string theory, and describe M-theory lift of this supersymmetry breaking type IIA brane configurations, along the line of [8, 9, 10, 11, 12, 13]. The role of flavor D4-branes, i.e., a misalignment of these D4-branes, is crucial to describe these brane configurations.

In section 4, we summarize what we have done in previous sections. We describe the similar application to the same $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with different matter contents, in the sense that the above symmetric flavor is replaced by eight fundamentals and an antisymmetric flavor for the $SU(N_c)$ gauge group. The theory given by the first gauge group $SU(N_c)$ with matters is based on the previous works of [14, 15, 16]. We also make some comments for the future directions.

2 The $\mathcal{N} = 1$ supersymmetric brane configurations

In order to study the meta-stable brane configuration, it is necessary to take two steps. One of them is to have nonzero masses for the quarks corresponding to relative displacement between D6-branes and D4-branes and the other is to take the Seiberg dual theory by standard brane motion. For the latter, we need to understand both brane configurations from the electric theory and the magnetic theory since the magnetic theory can be obtained from the electric theory [3]. These brane configurations corresponding to the gauge theory we are considering are not known so far in the literature and we describe them more explicitly. There exist two possible magnetic brane configurations, depending on whether the dual gauge group we take is the first gauge group or the second gauge group. Note that although the gauge group is a product gauge group, it is not always possible to take the dual for each of the gauge group independently, in the context of meta-stable brane configuration and for example, see [8].

2.1 Electric theory with $SU(N_c) \times SU(N'_c)$ gauge group

The gauge group we are interested in is given by $SU(N_c) \times SU(N'_c)$ and the matter contents are as follows:

- N_f -chiral multiplets Q are in the representation $(\mathbf{N}_c, \mathbf{1})$, and N_f -chiral multiplets \tilde{Q} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1})$, under the gauge group
- N'_f -chiral multiplets Q' are in the representation $(\mathbf{1}, \mathbf{N}'_c)$, and N'_f -chiral multiplets \tilde{Q}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}}'_c)$, under the gauge group
- The flavor-singlet field X is in the bifundamental representation $(\mathbf{N}_c, \overline{\mathbf{N}}'_c)$, and its conjugate field \tilde{X} is in the bifundamental representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c)$, under the gauge group
- The flavor-singlet field S , which is in a symmetric tensor representation under the $SU(N_c)$, is in the representation $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1})$, and its conjugate field \tilde{S} is in the representation $(\frac{1}{2}\overline{\mathbf{N}}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1})$, under the gauge group

If there are no symmetric and conjugate symmetric tensors, S and \tilde{S} , this theory is described by the work of [6, 4, 5] from field theory analysis or corresponding brane dynamics. Ignoring the presence of the fields Q', \tilde{Q}', X and \tilde{X} , then this theory will reduce to a single gauge group $SU(N_c)$ with a symmetric flavor, conjugate symmetric flavor and fundamental flavors S, \tilde{S}, Q and \tilde{Q} discussed in [6, 7, 11]. Now it is easy to check that the coefficient of the beta function of the first gauge group is given by

$$b_{SU(N_c)} = 3N_c - N_f - N'_c - (N_c + 2)$$

where the index of the symmetric representation of $SU(N_c)$ corresponding to S and \tilde{S} is equal to $\frac{1}{2}(N_c + 2)$. On the other hand, the coefficient of the beta function of the second gauge

group is given by

$$b_{SU(N'_c)} = 3N'_c - N'_f - N_c.$$

This theory is asymptotically free when the condition $b_{SU(N_c)} > 0$ is satisfied for the $SU(N_c)$ gauge group and when the condition $b_{SU(N'_c)} > 0$ is satisfied for the $SU(N'_c)$ gauge group. We'll see how these coefficients change in the magnetic theory.

The classical superpotential is given by

$$W = \mu A^2 + SA\tilde{S} + \lambda QA\tilde{Q} + \mu' A'^2 + \lambda' Q' A' \tilde{Q}' + XA\tilde{X} + \tilde{X}A'X + mQ\tilde{Q} + m'Q'\tilde{Q}', \quad (2.1)$$

where the coefficient functions are given by four rotation angles, which will be discussed in Figure 1, as follows

$$\mu \equiv \tan \theta, \quad \mu' \equiv \tan(\theta' - \theta), \quad \lambda \equiv \sin(\theta - \omega), \quad \lambda' \equiv \sin(\theta' - \theta - \omega').$$

Here the adjoint field for $SU(N_c)$ gauge group is denoted by A while the adjoint field for $SU(N'_c)$ gauge group is denoted by A' . The mass terms of these adjoint fields are related to the rotation angles of NS5-branes in type IIA brane configuration. The couplings of fundamentals with these adjoint fields are related also to the rotation angles of NS5-branes as well as the rotation angles of D6-branes in type IIA brane configuration. We add the mass terms for each fundamental flavor. The second term in (2.1) arises from the presence of a symmetric flavor S and a conjugate symmetric flavor \tilde{S} . Except this term and the last two mass terms, the above superpotential becomes the one studied in [15, 4]. Setting the fields $Q', \tilde{Q}', X, \tilde{X}$ and A' to zero, the superpotential becomes the one described in [7, 11]. After integrating out the adjoint fields A and A' , this superpotential (2.1) at $\theta = \frac{\pi}{2}$ and $\theta' = 0$ will reduce to the last two mass-deformed terms since the coefficient functions $\frac{1}{\mu}$ and $\frac{1}{\mu'}$ vanish at this particular rotation angles. It does not matter whether λ or λ' vanishes since eventhough these coefficient functions are not zero, λ - or λ' -dependent terms all vanish due to the $\frac{1}{\mu}$ factor or $\frac{1}{\mu'}$ factor. For the nonsupersymmetric brane configuration in section 3, we will consider this particular brane configuration with the constraint $\theta = \frac{\pi}{2}$ and $\theta' = 0$ all the time.

Then what is brane configuration for this gauge theory with given matter contents? It is known that the brane configuration for a single gauge group $SU(N_c)$ with a symmetric flavor, conjugate symmetric flavor and fundamental flavors S, \tilde{S}, Q and \tilde{Q} is represented by the work of [7, 11]: three NS5-branes, N_c D4-branes, $2N_f$ D6-branes and orientifold 6-plane where a middle NS5-brane is located. Now we add the extra two outer NS5-branes, in a \mathbf{Z}_2 symmetric way due to the O6-plane, to this brane configuration corresponding to the first gauge group

$SU(N_c)$ and we also put extra N'_c D4-branes and extra N'_f D6-branes for the second gauge group $SU(N'_c)$ (and their mirrors).

Then the type IIA brane configuration we are interested in consists of five NS5-branes, N_c - and N'_c - D4-branes suspended between them, $2N_f$ and $2N'_f$ D6-branes and orientifold 6 plane of positive RR charge. For the negative RR charge, the matter contents of S and \tilde{S} are replaced by an antisymmetric and conjugate antisymmetric flavors A and \tilde{A} , as in [7, 11]. Let us summarize the $\mathcal{N} = 1$ supersymmetric electric brane configuration we are studying in type IIA string theory as follows:

- First $NS5_{-\theta'}$ -brane (0123vw) with $x^6 < 0$
- Second $NS5_{\theta}$ -brane (0123vw) with $x^6 < 0$
- Third NS5-brane (012345) with $w = 0 = x^6$
- Fourth $NS5_{-\theta}$ -brane (0123vw) with $x^6 > 0$
- Fifth $NS5_{\theta'}$ -brane (0123vw) with $x^6 > 0$
- First N'_f D6 $_{-\omega'}$ -branes (01237vw) with $x^6 < 0$
- Second N_f D6 $_{\omega}$ -branes (01237vw) with $x^6 < 0$
- Third N_f D6 $_{-\omega}$ -branes (01237vw) with $x^6 > 0$
- Fourth N'_f D6 $_{\omega'}$ -branes (01237vw) with $x^6 > 0$
- O6-plane (0123789) with $v = 0 = x^6$
- N_c D4-branes (01236) with $v = 0 = w$
- N'_c D4-branes (01236) with $v = 0 = w$

Here we introduce two complex coordinates $v \equiv x^4 + ix^5$ and $w \equiv x^8 + ix^9$, as usual, and the worldvolume (vw) for the rotated branes above corresponds to the real 2-dimensions spanned in (v, w) plane. The mirrors are located in a \mathbf{Z}_2 symmetric way. The N_c D4-branes are suspended between $NS5_{\theta}$ -brane and $NS5_{-\theta}$ -brane while the N'_c D4-branes are suspended between $NS5_{-\theta'}$ -brane and $NS5_{\theta'}$ -brane(and their mirrors). The convention for the rotated branes is the same as the one used in [11, 10, 8].

Let us draw the type IIA brane configuration we describe in Figure 1 and we put N_f D6 $_{-\omega}$ -branes and N'_f D6 $_{\omega'}$ -branes in the nonzero v direction for nonzero mass terms for the fundamentals(and their mirrors). If we are detaching $NS5_{\pm\theta}$ -branes, D6 $_{\pm\omega}$ -branes and N'_c D4-branes(and its mirrors), then this brane configuration will reduce to the one described in [7, 11]. If we are detaching a middle NS5-brane to the x^7 direction, then this will lead to the brane configuration considered in [17, 8] with the gauge group $SO(N_c) \times SU(N'_c)$ with fundamentals for each gauge group and bifundamentals. With O6-plane of negative RR charge instead of having positive RR charge, this process will lead to the gauge group $Sp(N_c) \times SU(N'_c)$ with fundamentals for each gauge group and bifundamentals analyzed in

[17, 8]. If we are detaching all the branes living on the negative x^6 region and O6-plane, then this will become the brane configuration described in the work of [4, 15].

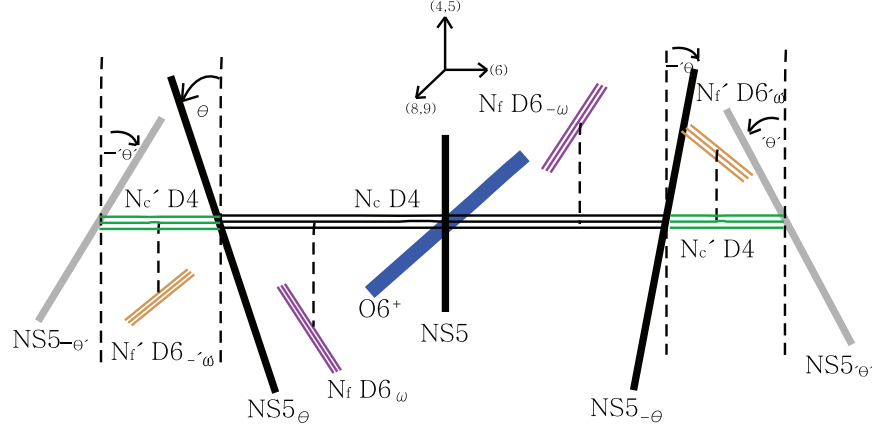


Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration with $SU(N_c) \times SU(N'_c)$ gauge group with fundamentals Q, \tilde{Q}, Q' and \tilde{Q}' for each gauge group, the bifundamentals X and \tilde{X} and a symmetric flavor S and a conjugate symmetric flavor \tilde{S} for $SU(N_c)$.

2.2 Magnetic theory with $SU(\tilde{N}_c) \times SU(N'_c)$ gauge group

By brane motion, one gets the Seiberg dual theory for the gauge group $SU(N_c)$. From the magnetic brane configuration which is shown in Figure 2 that is obtained by interchanging a set of $NS5_\theta$ -brane and $D6_\omega$ -branes and a set of $NS5_{-\theta}$ -brane and $D6_{-\omega}$ -branes each other, the linking number [18] of $NS5_{\theta=\frac{\pi}{2}}$ -brane can be computed and is $L_5 = \frac{N_f}{2} - \tilde{N}_c + N_f + N'_c$ when the N_f $D6$ -branes are parallel to a middle $NS5$ -brane, as in the situation of [11]. On the other hand, the linking number of $NS5_{\theta=\frac{\pi}{2}}$ -brane from the electric brane configuration in Figure 1 can be read off and is given by $L_5 = -\frac{N_f}{2} + N_c - N'_c$. Then the number of dual color \tilde{N}_c , by linking number conservation, is given by

$$\tilde{N}_c = 2(N_f + N'_c) - N_c. \quad (2.2)$$

Compared with the Figure 1, the magnetic brane configuration in Figure 2 has several different features due to the change of the locations for the branes corresponding to the gauge group $SU(N_c)$. Between the first $NS5_{-\theta'}$ -brane and second $NS5_{-\theta}$ -brane, there are extra N_f $D6_{-\omega}$ -branes and newly created $2N_f$ flavor D4-branes connecting the second $NS5_{-\theta}$ -brane and $D6_{-\omega}$ -branes (and their mirrors) when $\theta \neq \frac{\pi}{2}$, implying that the matter contents for the gauge group $SU(N'_c)$ will change and this will affect the computation for the coefficient of beta function below. Therefore, these features will provide the various interaction terms in the dual magnetic superpotential we will describe later.

Then the dual magnetic gauge group is $SU(\tilde{N}_c) \times SU(N'_c)$ with (2.2) and the matter contents are as follows:

- N_f -chiral multiplets q are in the representation $(\tilde{\mathbf{N}}_c, 1)$, N_f -chiral multiplets \tilde{q} are in the representation $(\overline{\tilde{\mathbf{N}}_c}, 1)$, under the gauge group
- N'_f -chiral multiplets Q' are in the representation $(\mathbf{1}, \mathbf{N}'_c)$, and N'_f -chiral multiplets \tilde{Q}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}'_c})$, under the gauge group
- The flavor-singlet field Y is in the bifundamental representation $(\tilde{\mathbf{N}}_c, \overline{\mathbf{N}'_c})$, and its complex conjugate field \tilde{Y} is in the bifundamental representation $(\overline{\tilde{\mathbf{N}}_c}, \mathbf{N}'_c)$, under the gauge group
- The flavor-singlet field s , which is in a symmetric tensor representation under the $SU(\tilde{N}_c)$, is in the representation $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c + \mathbf{1}), \mathbf{1})$, and its conjugate field \tilde{s} is in the representation $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c + \mathbf{1}), \mathbf{1})$, under the gauge group

There are also $(N_f + N'_c)^2$ gauge-singlets in the first dual gauge group factor as follows:

- N_f -fields F' are in the representation $(\mathbf{1}, \mathbf{N}'_c)$, and its complex conjugate N_f -fields \tilde{F}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}'_c})$, under the gauge group
 - N_f^2 -fields M' are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
 - The $N_c'^2$ -fields Φ' is in the representation $(\mathbf{1}, \mathbf{N}_c'^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the gauge group
- Moreover, there are additional $N_f(2N_f + 1)$ gauge-singlets
- N_f^2 -fields N' are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
 - $\frac{1}{2}N_f(N_f + 1)$ -fields P' are in the representation $(\mathbf{1}, \mathbf{1})$, and its conjugate $\frac{1}{2}N_f(N_f + 1)$ -fields \tilde{P}' are in the representation $(\mathbf{1}, \mathbf{1})$, under the gauge group

More explicitly, these are represented by $N' \equiv Q\tilde{S}S\tilde{Q}$, $P' \equiv Q\tilde{S}Q$ and $\tilde{P}' \equiv \tilde{Q}S\tilde{Q}$ in terms of fields in electric theory explained in [6, 11]. Although these gauge singlets appear in the dual magnetic superpotential for the general rotation angles θ and θ' , the case $\theta = \frac{\pi}{2}$ we are considering does not contain these gauge singlets, as observed in [11].

The coefficient of the beta function of the first dual gauge group factor, as done in electric theory, is given by

$$b_{SU(\tilde{N}_c)}^{mag} = 3\tilde{N}_c - N_f - N'_c - (\tilde{N}_c + 2)$$

and the coefficient of the beta function of the second gauge group factor is given by

$$b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - \tilde{N}_c - N_f - N'_c.$$

Then both $SU(\tilde{N}_c)$ and $SU(N'_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One relies on the perturbative calculations at low energy for this magnetic IR free region with $b_{SU(\tilde{N}_c)}^{mag} < 0$ and $b_{SU(N'_c)}^{mag} < 0$. It is clear, from the magnetic

and electric brane configurations in Figure 2 and Figure 1, that the $SU(N'_c)$ fields in the magnetic theory are different from those of the electric theory ¹.

The dual magnetic superpotential ² for massless fundamental flavors Q' and \widetilde{Q}' (i.e., $m' = 0$) and massive fundamental flavors Q and \widetilde{Q} is given by

$$W_{dual} = (M' q \widetilde{s} s \widetilde{q} + m M') + \widetilde{Y} \widetilde{F}' q + Y \widetilde{q} F' + \Phi' Y \widetilde{Y} + (\Phi'^2 + \dots) \quad (2.3)$$

where the mesons are given in terms of fields in the electric theory (See also the relevant works found in [9, 8])

$$M' \equiv Q \widetilde{Q}, \quad F' \equiv \widetilde{X} Q, \quad \widetilde{F}' \equiv X \widetilde{Q}, \quad \Phi' \equiv X \widetilde{X}.$$

Here the last piece $(\Phi'^2 + \dots)$ in W_{dual} above is coming from the superpotential (2.1) in an electric theory and contains also $N' q \widetilde{q} + P' q \widetilde{s} q + \widetilde{P}' \widetilde{q} s \widetilde{q}$ for the general rotation angles θ, θ', ω

¹More explicitly the conditions $b_{SU(\widetilde{N}_c)}^{mag} < 0$ and $b_{SU(N_c)} > 0$ imply that $N_f + N'_c < \frac{2}{3}N_c + \frac{2}{3}$. Also the number of dual colors \widetilde{N}_c defined as (2.2) should be positive. Then the range for the N_f in the first gauge group can be written as $\frac{1}{2}N_c < N_f + N'_c < \frac{2}{3}N_c + \frac{2}{3}$. Since $b_{SU(N'_c)} - b_{SU(N_c)}^{mag} = 3(N'_c + N_f) - 2N_c < 0$, if we require that $b_{SU(N'_c)}^{mag} < 0$ which is equivalent to $N_c - 3N_f < N'_c$, then the electric description of $SU(N'_c)$ is IR free because $b_{SU(N'_c)} < 0$. At high energy, $SU(N'_c)$ theory is strongly coupled while $SU(N_c)$ theory is UV free. At the scale Λ_1 , the $SU(N_c)$ theory is strongly coupled and the Seiberg duality occurs. All the running couplings are changed by this duality and all the coefficients of beta functions, $b_{SU(\widetilde{N}_c)}^{mag}$ and $b_{SU(N'_c)}^{mag}$ become negative. Then at energy scale lower than Λ_1 , the theory is weakly coupled. When $b_{SU(N'_c)}^{mag} < b_{SU(\widetilde{N}_c)}^{mag} < 0$, the one loop computation is reliable with $\Lambda_1 \ll \Lambda_2$. When $b_{SU(\widetilde{N}_c)}^{mag} < b_{SU(N'_c)}^{mag} < 0$, the requirement that $SU(N'_c)^{mag}$ theory is less coupled than the $SU(\widetilde{N}_c)^{mag}$ at the supersymmetry breaking scale μ provides a stronger constraint on Λ_2 . It is not enough to choose it higher than Λ_1 simply. Then under the constraint, $\Lambda_2 \gg \left(\frac{\Lambda_1}{\mu}\right)^b \Lambda_1$ where $b \equiv \frac{b_{SU(\widetilde{N}_c)}^{mag} - b_{SU(N'_c)}^{mag}}{b_{SU(N'_c)}} > 0$, one can ignore the contribution from the gauge coupling of $SU(N'_c)^{mag}$ at the supersymmetry breaking scale and one relies on the one loop computation. See the ref. [19] for the relevant discussions in the context of quiver gauge theory. In particular, the appendix B of [19].

² Although gauging the $SU(\widetilde{N}_c)$ does not affect the supersymmetry breaking vacua which will be discussed in next section, it leads to the supersymmetry vacua. We integrate out the bifundamentals Y and \widetilde{Y} in such a way that the gauge group $SU(\widetilde{N}_c)$ is not broken by the fields Y and \widetilde{Y} , as in meta-stable state, so $\langle Y \rangle = 0 = \langle \widetilde{Y} \rangle$ in (3.1). For nonzero vacuum expectation values for M' , this superpotential gives the $SU(\widetilde{N}_c)$ “flavors” $q \widetilde{s}$ and $s \widetilde{q}$, the mass $\langle M' \rangle$. Below the energy scale $\langle M' \rangle$, one can integrate out these massive flavors using the equations of motion $\langle q \widetilde{s} \rangle = 0 = \langle s \widetilde{q} \rangle$. Then the low energy theory is given by $SU(\widetilde{N}_c)$ pure Yang-Mills theory and the corresponding scale matching condition connecting between the low energy scale Λ_L and the macroscopic scale $\widetilde{\Lambda}$ can be computed. Then the low energy theory has a superpotential term which is proportional to $\left(\widetilde{\Lambda}^{2\widetilde{N}_c - N_f - N'_c - 2} \det M'\right)^{\frac{1}{\widetilde{N}_c}}$ plus $m M'$. Using this dynamically generated superpotential the vacuum expectation value for M' is obtained in the supersymmetric vacuum. There is no conserved $U(1)_R$ symmetry because it is anomalous under the gauged $SU(\widetilde{N}_c)$ in the sense that the determinant term above breaks it explicitly. Therefore, the $U(1)_R$ symmetry returns an “approximate” accidental symmetry of the IR theory. See the ref. [20] for the discussion on the relation between the R-symmetry breaking and supersymmetry breaking and also the refs. [2, 21] on the recent revival on this subject. The small parameter of [21] corresponds to the above $\widetilde{\Lambda}^{(2\widetilde{N}_c - N_f - N'_c - 2)/\widetilde{N}_c}$ with negative exponent.

and ω' . When both $\theta = \frac{\pi}{2}$ and $\theta' = 0$, this piece will vanish and the superpotential consists of the first five terms in (2.3) that are relevant part for the meta-stable brane configuration next section. The strings stretching between the N_f $D6_\omega$ -branes and N'_c D4-branes lead to the gauge theory objects for the additional N_f $SU(N'_c)$ fundamentals F' and the additional N_f $SU(N'_c)$ antifundamentals $\widetilde{F'}$. The fluctuations of the singlet Φ' correspond to the motion of N'_c D4-branes suspended two NS5-branes (and its mirrors). The fluctuations of the singlet M' correspond to the motion of additional N_f -flavor D4-branes suspended between D6-branes and NS5-brane (and its mirrors).

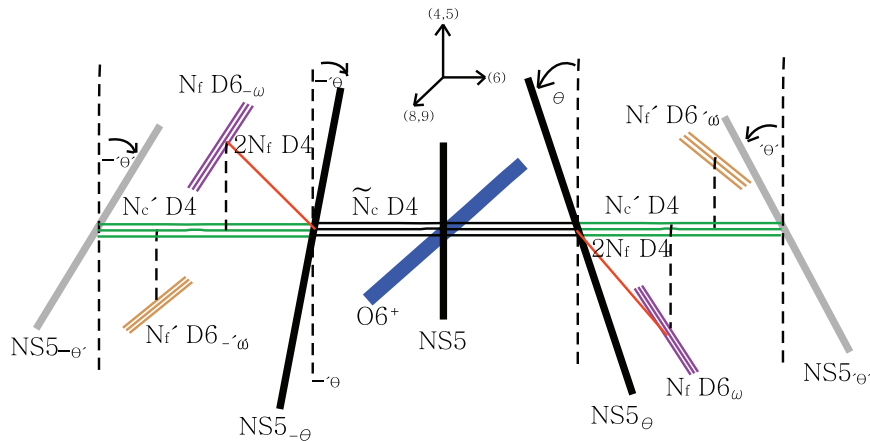


Figure 2: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $SU(\widetilde{N}_c = 2N_f + 2N'_c - N_c) \times SU(N'_c)$ gauge group with fundamentals q, \widetilde{q}, Q' and \widetilde{Q}' for each gauge group, the bifundamentals Y and \widetilde{Y} and a symmetric flavor s and a conjugate symmetric flavor \widetilde{s} for $SU(\widetilde{N}_c)$ and various gauge singlets. When $\theta = \frac{\pi}{2}$, the number of newly created flavor D4-branes connecting $D6_\omega$ -branes and $NS5_{\theta=\frac{\pi}{2}}$ is reduced to N_f , not $2N_f$ as observed in [11]. Compared with the electric brane configuration in Figure 1, the second NS5-brane with N_f D6-branes and the fourth NS5-brane with other N_f D6-branes are interchanged along x^6 -direction each other.

2.3 Magnetic theory with $SU(N_c) \times SU(\tilde{N}'_c)$ gauge group

In this subsection, we consider the other magnetic theory. By brane motion, one gets the Seiberg dual theory for the second gauge group $SU(N'_c)$. From the magnetic brane configuration which is shown in Figure 3 obtained by interchanging a set of $NS5_{\theta'}$ -brane and $D6_{\omega'}$ -branes and $NS5_{-\theta}$ -brane, the linking number of $NS5_{-\theta=\frac{\pi}{2}}$ -brane is given by $L_5 = \frac{N'_f}{2} - \tilde{N}'_c$ as long as $D6_{\omega'}$ -branes are not parallel to $NS5_{-\theta}$ -brane. Moreover, the linking number of $NS5_{-\theta=\frac{\pi}{2}}$ -brane from the electric brane configuration in Figure 1 is given by $L_5 = -\frac{N'_f}{2} + N'_c - N_c$. Then the number of dual color \tilde{N}_c , by linking number conservation, is

given by

$$\tilde{N}'_c = N'_f + N_c - N'_e. \quad (2.4)$$

Compared with the Figure 1, the magnetic brane configuration in Figure 3 has several different features. Between the second $NS5_{-\theta'}$ -brane and third NS5-brane, there are extra N'_f $D6_{-\omega'}$ -branes and newly created N'_f D4-branes connecting the second $NS5_{-\theta'}$ -brane and $D6_{-\omega'}$ -branes (and their mirrors), implying that the $SU(N_c)$ fields will change and affect for the computation of coefficient of beta function below. So this will provide the interaction terms, that did not appear in the electric theory, in the dual magnetic superpotential.

Then the dual magnetic gauge group is $SU(N_c) \times SU(\tilde{N}'_c)$ with (2.4) and the matter contents are as follows:

- N_f -chiral multiplets Q are in the representation $(\mathbf{N}_c, \mathbf{1})$, and N_f -chiral multiplets \tilde{Q} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1})$, under the gauge group
- N'_f -chiral multiplets q' are in the representation $(\mathbf{1}, \tilde{\mathbf{N}}'_c)$, N'_f -chiral multiplets \tilde{q}' are in the representation $(\mathbf{1}, \overline{\tilde{\mathbf{N}}}'_c)$, under the gauge group
- The flavor-singlet field Y is in the bifundamental representation $(\mathbf{N}_c, \overline{\tilde{\mathbf{N}}}'_c)$, and its complex conjugate field \tilde{Y} is in the bifundamental representation $(\overline{\mathbf{N}}_c, \tilde{\mathbf{N}}'_c)$, under the gauge group
- The flavor-singlet field S , which is in a symmetric tensor representation under the $SU(N_c)$, is in the representation $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1})$, and its conjugate field \tilde{S} is in the representation $(\frac{1}{2}\overline{\mathbf{N}}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1})$, under the gauge group

There are $(N'_f + N_c)^2$ gauge-singlets in the second dual gauge group factor as follows:

- N'_f -fields F are in the representation $(\mathbf{N}_c, \mathbf{1})$, and its complex conjugate N'_f -fields \tilde{F} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1})$, under the gauge group
- N'^2_f -fields M are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
- The N_c^2 -fields Φ is in the representation $(\mathbf{N}_c^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the gauge group

The coefficient of the beta function of the first gauge group factor is given by

$$b_{SU(N_c)}^{mag} = 3N_c - N_f - \tilde{N}'_c - N'_f - N_c - (N_c + 2)$$

and the coefficient of the beta function of the second gauge group factor is given by

$$b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c.$$

It is evident that the $SU(N_c)$ fields in the magnetic theory in Figure 3 are different from those of the electric theory in Figure 1³. Since $b_{SU(N_c)} - b_{SU(N_c)}^{mag} > 0$, the $SU(N_c)$ theory is more asymptotically free than the $SU(N_c)^{mag}$ theory [5].

³Now the conditions $b_{SU(\tilde{N}'_c)}^{mag} < 0$ and $b_{SU(N'_c)} > 0$ imply that $N'_f + N_c < \frac{3}{2}N'_c$. Also the number of dual

The dual magnetic superpotential ⁴ for massless fundamental flavors Q and \tilde{Q} (i.e., $m = 0$) and massive fundamental flavors Q' and \tilde{Q}' is

$$W'_{dual} = \left(M q' \tilde{q}' + m' M \right) + Y \tilde{F} q' + \tilde{Y} q' F + \Phi Y \tilde{Y} + (\Phi^2 + \dots) \quad (2.5)$$

where the mesons are given by

$$M \equiv Q' \tilde{Q}', \quad F \equiv X Q', \quad \tilde{F} \equiv \tilde{X} \tilde{Q}', \quad \Phi \equiv X \tilde{X}.$$

Here the last piece $(\Phi^2 + \dots)$ in W_{dual} is coming from the superpotential (2.1) in an electric theory for the general rotation angles θ, θ', ω and ω' . When $\theta = \frac{\pi}{2}$ and $\theta' = 0$, this will vanish and the superpotential consists of the first five terms in (2.5). As we observed, the presence of N'_f $D6_{\omega'}$ -branes and N'_f D4-branes give rise to the gauge-singlets. That is, the strings stretching between the N'_f $D6_{\omega'}$ -branes and N_c D4-branes lead to the additional N'_f $SU(N_c)$ fundamentals F and additional N'_f $SU(N_c)$ antifundamentals \tilde{F} . The fluctuations of the singlet Φ correspond to the motion of N_c D4-branes suspended two NS5-branes (and its mirrors). The fluctuations of the singlet M correspond to the motion of additional N'_f flavor D4-branes suspended between D6-branes and NS5-brane (and its mirrors).

colors \tilde{N}'_c defined as (2.4) should be positive. Then the range for the N'_f in the second gauge group can be written as $N'_c < N'_f + N_c < \frac{3}{2} N'_c$. The condition $b_{SU(N_c)}^{mag} < 0$ implies $N'_c - 2N'_f - 2 < N_f$. The $b_{SU(N_c)}$ can be IR free or UV free in the electric description. If the former where $b_{SU(N_c)} < 0$ happens, then one can analyze the method in the footnote 1 exactly. When $b_{SU(N_c)}^{mag} < b_{SU(\tilde{N}'_c)}^{mag} < 0$, the one loop computation is reliable with

$\Lambda_2 \ll \Lambda_1$. When $b_{SU(\tilde{N}'_c)}^{mag} < b_{SU(N_c)}^{mag} < 0$, under the constraint, $\Lambda_1 \gg \left(\frac{\Lambda_2}{\mu} \right)^b \Lambda_2$ where $b \equiv \frac{b_{SU(\tilde{N}'_c)}^{mag} - b_{SU(N_c)}^{mag}}{b_{SU(N_c)}}$, one can ignore the contribution from the gauge coupling of $SU(N_c)^{mag}$ at the supersymmetry breaking scale and one relies on the one loop computation. If the latter happens where $b_{SU(N_c)} > 0$, then for the case where the $SU(N_c)^{mag}$ theory becomes more IR free than the $SU(\tilde{N}'_c)^{mag}$, in other words, $b_{SU(N_c)}^{mag} < b_{SU(\tilde{N}'_c)}^{mag} < 0$ after

Seiberg duality the coupling of the $SU(N_c)^{mag}$ becomes more smaller than the coupling of $SU(\tilde{N}'_c)^{mag}$ along the flow to the low energy. Then the one loop computation is reliable with $\Lambda_1 \ll \Lambda_2$. For the case where the $SU(N_c)^{mag}$ theory becomes less IR free than the $SU(\tilde{N}'_c)^{mag}$, in other words, $b_{SU(\tilde{N}'_c)}^{mag} < b_{SU(N_c)}^{mag} < 0$, under

the strong constraint, $\Lambda_1 \ll \left(\frac{\Lambda_2}{\mu} \right)^b \Lambda_2 \ll \Lambda_2$ where b is the same as above, one can ignore the contribution from the gauge coupling of $SU(N_c)^{mag}$ at the supersymmetry breaking scale and one relies on the one loop computation.

⁴ Although gauging the $SU(\tilde{N}'_c)$ does not affect the supersymmetry breaking vacua which will be discussed in next section, it leads to the supersymmetry vacua. For nonzero vacuum expectation values for M , this superpotential gives the $SU(\tilde{N}'_c)$ fundamental flavors q' and \tilde{q}' , the mass $< M >$. Below the energy scale $< M >$, one can integrate out these massive flavors using the equations of motion $< q' > = 0 = < \tilde{q}' >$. Then the low energy theory is given by $SU(\tilde{N}'_c)$ pure Yang-Mills theory and the corresponding scale matching condition connecting between the low energy scale Λ_L and the macroscopic scale $\tilde{\Lambda}$ can be computed. Then the low energy theory has a superpotential term which is proportional to $\left(\tilde{\Lambda}^{3\tilde{N}'_c - N'_f - N_c} \det M \right)^{\frac{1}{\tilde{N}'_c}}$ plus $m' M$. There is no conserved $U(1)_R$ symmetry because it is anomalous under the gauged $SU(\tilde{N}'_c)$ in the sense that

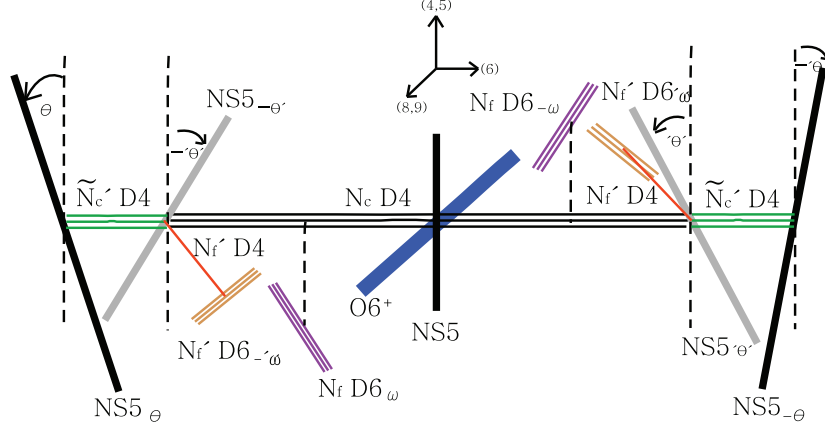


Figure 3: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c - N'_c)$ gauge group with fundamentals Q, \tilde{Q}, q' and \tilde{q}' for each gauge group, the bifundamentals Y and \tilde{Y} and a symmetric flavor S and a conjugate symmetric flavor \tilde{S} for $SU(N_c)$ and various gauge singlets. Compared with Figure 1, the first NS5-brane and the second NS5-brane with N'_f D6-branes are interchanged along x^6 direction each other (and its mirrors).

3 Nonsupersymmetric meta-stable brane configuration

Based on the magnetic brane configurations we have found in previous section, we describe the nonsupersymmetric brane configurations by recombination of flavor D4-branes and color D4-branes and splitting between those flavor D4-branes and the remnant of flavor D4-branes which does not participate in the recombination process.

3.1 When the magnetic gauge group is $SU(\tilde{N}_c) \times SU(N'_c)$

In this case, the dual magnetic superpotential is given by the first five terms of (2.3) at $\theta = \frac{\pi}{2}$ and $\theta' = 0$. The dual quarks q and \tilde{q} are fundamental $(\tilde{\mathbf{N}}_c, \mathbf{1})$ and antifundamental $(\overline{\tilde{\mathbf{N}}_c}, \mathbf{1})$ for the gauge group indices and antifundamentals for the flavor indices. The flavor-singlet fields s and \tilde{s} are symmetric $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c + \mathbf{1}), \mathbf{1})$ and conjugate symmetric tensor $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c + \mathbf{1}), \mathbf{1})$ for the gauge group indices respectively. The quantity $q\tilde{s}s\tilde{q}$ has a rank \tilde{N}_c and the mass matrix m has a rank N_f . Then the F-term equation for M' cannot be satisfied if the rank N_f exceeds \tilde{N}_c and the supersymmetry is broken.

the determinant term above breaks it explicitly. Remember that the R charges for the fields are as follows: $R(Y) = R(q') = \frac{N'_c}{N'_f + N_c}$ and $R(M) = R(F) = R(\Phi) = 2 - \frac{2N'_c}{N'_f + N_c}$. Therefore, the $U(1)_R$ symmetry returns an “approximate” accidental symmetry of the IR theory.

The classical moduli space of vacua can be obtained from F-term equations and one gets

$$\begin{aligned}
q\tilde{s}s\tilde{q} + m &= 0, & \tilde{s}s\tilde{q}M' + \tilde{Y}\tilde{F}' &= 0, \\
s\tilde{q}M'q &= 0, & \tilde{q}M'q\tilde{s} &= 0, \\
M'q\tilde{s}s + F'Y &= 0, & \tilde{F}'q + \Phi'Y &= 0, \\
q\tilde{Y} &= 0, & \tilde{q}F' + \tilde{Y}\Phi' &= 0, \\
Y\tilde{q} &= 0, & Y\tilde{Y} &= 0.
\end{aligned}$$

Some of F-term equations are satisfied if one takes the zero vacuum expectation values for the fields Y, \tilde{Y}, F' and \tilde{F}' . Then, it is easy to see that

$$s\tilde{q}M' = 0 = M'q\tilde{s}, \quad q\tilde{s}s\tilde{q} + m = 0.$$

Then the solutions can be written as

$$\begin{aligned}
\langle q\tilde{s} \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \langle s\tilde{q} \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{pmatrix} \\
\langle Y \rangle &= \langle \tilde{Y} \rangle = \langle F' \rangle = \langle \tilde{F}' \rangle = 0.
\end{aligned} \tag{3.1}$$

Let us expand around on a point on (3.1), as done in [1]. Then the remaining relevant terms of superpotential are given by

$$W_{dual}^{rel} = \Phi_0 \left(\delta\hat{\varphi} \delta\hat{\tilde{\varphi}} + m \right) + \delta Z \delta\hat{\varphi} s_0 \tilde{q}_0 + \delta\tilde{Z} q_0 \tilde{s}_0 \delta\hat{\tilde{\varphi}}$$

by following the fluctuations for the various fields in [11]. Note that there exist three kinds of terms, the vacuum $\langle q \rangle$ multiplied by $\delta\tilde{Y}\delta\tilde{F}'$, the vacuum $\langle \tilde{q} \rangle$ multiplied by $\delta F'\delta Y$, and the vacuum $\langle \Phi' \rangle$ multiplied by $\delta Y\delta\tilde{Y}$. By redefining these as $\delta\hat{\tilde{Y}}\delta\hat{\tilde{F}'}$, $\delta\hat{F}'\delta\hat{Y}$, and $\delta\hat{Y}\delta\hat{\tilde{Y}}$ respectively, they do not enter the contributions for the one loop result, up to quadratic order. As done in [11], the defining function $\mathcal{F}(v^2)$ can be computed and using the equation (2.14) of [22] of $m_{\Phi_0}^2$ and $\mathcal{F}(v^2)$, one gets that $m_{\Phi_0}^2$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

Let us recombine \tilde{N}_c flavor D4-branes among the additional N_f flavor D4-branes with those connecting NS5'-brane(coming from $NS5_{-\theta}$ -brane) and NS5-brane and then push them in the $+v$ direction from the magnetic brane configuration in Figure 2, as done in [23, 24, 25]. Due to the presence of middle NS5-brane, this procedure, pushing into the $+v$ direction, is possible. This is different feature, compared with the one in [8] where there was no meta-stable brane configuration, when we take the Seiberg dual for the gauge group $SO(N_c)$, since there was no extra NS5-brane, unlike to the present case. Of course, the mirrors will move

$-v$ direction due to the presence of O6-plane. There are no color D4-branes connecting NS5'-brane and NS5-brane and there exist only $(N_f - \tilde{N}_c)$ flavor D4-branes connecting D6-branes and NS5'-brane(and their mirrors) that are misaligned to the above \tilde{N}_c flavor D4-branes.

Then, the minimal energy supersymmetry breaking brane configuration is given by Figure 4 where one sees a misalignment between the additional flavor D4-branes. If we are detaching N'_c D4-branes, N'_f D6-branes and NS5-brane(coming from $NS5_{\theta'}$ -brane)(and its mirrors), then this brane configuration leads to the one described in [11]. If we are detaching all the branes living on the positive x^6 region and O6-plane, then this will look like the brane configuration of [8] with the product gauge group of unitary group shown in Figure 3 of [8]. The difference between these two appears in the the left NS5-brane: in Figure 4, it is NS5-brane while in Figure 3 of [8], it is given by NS5'-brane.

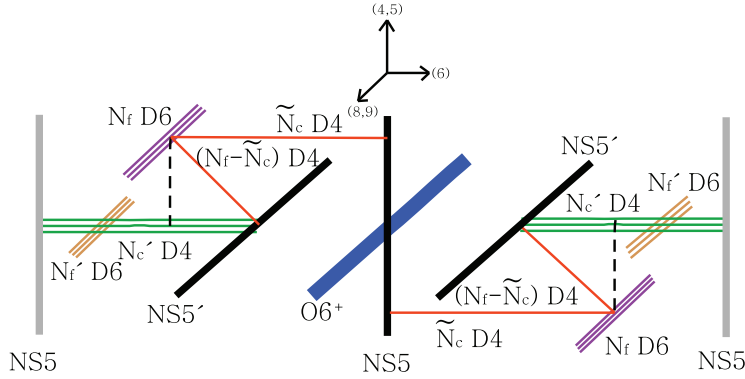


Figure 4: The nonsupersymmetric minimal energy brane configuration of $SU(\tilde{N}_c = 2N_f + 2N'_c - N_c) \times SU(N'_c)$ gauge group with fundamentals q, \tilde{q}, Q' and \tilde{Q}' for each gauge group, the bifundamentals Y and \tilde{Y} and a symmetric flavor s and a conjugate symmetric flavor \tilde{s} for $SU(\tilde{N}_c)$ and various gauge singlets. We consider the massless case of Q' and \tilde{Q}' . Compared with Figure 2, there is a misalignment of the additional N_f flavor D4-branes. Some of them are connecting to NS5'-brane and the other to NS5-brane. In this figure, the rotation angle θ of Figure 2 is $\frac{\pi}{2}$ while the rotation angle θ' is 0.

In [26, 27], the Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of $xt = (-1)^{N_f + N'_f} v^4 \prod_{k=1}^{N_f} (v^2 - e_k^2) \prod_{l=1}^{N'_f} (v^2 - e_l^2)$ was found. Since we are dealing with five NS5-branes, the magnetic M5-brane configuration in Figure 2 with equal mass for Q and \tilde{Q} and massless for Q' and \tilde{Q}' can be characterized by

the following quintic equation for t as follows:

$$t^5 + \left[v^{N'_c} \right] t^4 + \left[v^{\tilde{N}_c + N'_f} (v - m)^{N_f} \right] t^3 + \left[(-1)^{\tilde{N}_c} v^{\tilde{N}_c + 2N'_f + 2} (v - m)^{2N_f} \right] t^2 \\ + \left[(-1)^{N'_c} v^{N'_c + 3N'_f + 6} (v - m)^{3N_f} \right] t + \left[(-1)^{N_f + N'_f} v^{10 + 5N'_f} (v - m)^{4N_f} (v + m)^{N_f} \right] = 0.$$

The polynomial $g_1(v)$ [27] appearing in the coefficient t^4 in v of degree N'_c given by the number of D4-branes suspended between the first and second NS5-branes from Figure 4 has a highest power N'_c of v . The polynomial $g_4(v)$ appearing in the coefficient t in v of degree N'_c given by the number of D4-branes suspended between the fourth and fifth NS5-branes from Figure 4 can be expressed in terms of $g_1(v)$ with a replacement $v \rightarrow -v$. Similarly, the polynomial $g_2(v)$ appearing in the coefficient t^3 in v of degree \tilde{N}_c given by the number of D4-branes suspended between the second and third NS5-branes from Figure 2 has a highest power \tilde{N}_c of v and the polynomial $g_3(v)$ appearing in the coefficient t^2 in v of degree \tilde{N}_c given by the number of D4-branes suspended between the third and fourth NS5-branes from Figure 2 can be written as $g_2(v)$ with a replacement $v \rightarrow -v$. We also used the symmetry in other polynomials $J_i(v)$ where $i = 1, 2, 3, 4$ representing the contribution to the above space relating to the complex variables x, t and v from D6-brane charge sources, i.e., D6-branes and O6-plane between i -th and $(i + 1)$ -th NS5-branes. Note that $J_2(v) = J_3(v) = v^2$ and $\prod_{i=1}^4 J_i(v) = (-1)^{N_f + N'_f} v^{2N'_f + 4} (v^2 - m^2)^{N_f}$. Moreover, there exist relations $J_3(v) = J_2(-v)$ and $J_4(v) = J_1(-v)$ due to the \mathbf{Z}_2 symmetry of O6-plane.

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Now the asymptotic regions of various NS5-branes can be determined by reading off the first two terms of the quintic curve above giving the $NS5_L$ -brane asymptotic region, next two terms giving $NS5'_L$ -brane asymptotic region, next two terms giving $NS5_M$ -brane asymptotic region, next two terms giving $NS5'_R$ -brane asymptotic region, and final two terms giving $NS5_R$ -brane asymptotic region. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{N'_c} + \dots & NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{N_f + N'_f + 2} + \dots & NS5_M \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{2N_f + 2N'_f - N'_c + 4} + \dots & NS5_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v &\rightarrow +m, & y &\sim w^{\tilde{N}_c - N'_c + N_f + N'_f} + \dots & NS5'_L \text{ asymptotic region,} \\ v &\rightarrow -m, & y &\sim w^{N'_c - \tilde{N}_c + N_f + N'_f + 4} + \dots & NS5'_R \text{ asymptotic region.} \end{aligned}$$

The two $NS5'_{L,R}$ -branes are moving in the $\pm v$ direction respectively holding everything else fixed instead of moving D6-branes in the $\pm v$ direction. The corresponding mirrors of D4-branes are moved appropriately. The harmonic function in the Tau-NUT space, sourced by $2N_f$ D6-branes, O6-plane and $2N'_f$ D6-branes, can be determined once we fix the x^6 position for these branes. Then the first order differential equation for the $g(s)$ [25] where the absolute value of $g(s)$ is equal to the absolute value of w can be solved exactly with the appropriate boundary conditions on $NS5'_L$ or $NS5'_R$ asymptotic region from above classification 2. Since the extra terms in the harmonic function contribute to the $g(s)$ as a multiplication factor, the contradiction with the correct statement that y should vanish only if $v = 0$, implies that there exists the instability from a new M5-brane mode at some point from the transition of SQCD-like theory description to M-theory description.

3.2 When the magnetic gauge group is $SU(N_c) \times SU(\tilde{N}'_c)$

In this case, the dual magnetic superpotential is given by the first five terms of (2.5) at $\theta = \frac{\pi}{2}$ and $\theta' = 0$. The dual quarks q' and \tilde{q}' are fundamental $(\mathbf{1}, \tilde{\mathbf{N}}'_c)$ and antifundamental $(\mathbf{1}, \overline{\tilde{\mathbf{N}}}'_c)$ for the gauge group indices and antifundamentals for the flavor indices. The quantity $q'\tilde{q}'$ has a rank \tilde{N}'_c and the mass matrix m' has a rank N'_f . Then the F-term equation for M cannot be satisfied if the rank N'_f exceeds \tilde{N}'_c and the supersymmetry is broken.

The classical moduli space of vacua can be obtained from F-term equations and one gets

$$\begin{aligned} q'\tilde{q}' + m' &= 0, & \tilde{q}'M + Y\tilde{F} &= 0, \\ Mq' + F\tilde{Y} &= 0, & \tilde{F}q' + \tilde{Y}\Phi &= 0, \\ q'Y &= 0, & \tilde{q}'F + \Phi Y &= 0, \\ \tilde{Y}\tilde{q}' &= 0, & Y\tilde{Y} &= 0. \end{aligned}$$

Other F-term equations are satisfied if one takes the zero vacuum expectation values for the fields Y, \tilde{Y}, F and \tilde{F} . Then, it is easy to see that

$$\tilde{q}'M = 0 = Mq', \quad q'\tilde{q}' + m' = 0.$$

Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m}e^{\phi}\mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi}\mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0\mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix} \\ \langle Y \rangle &= \langle \tilde{Y} \rangle = \langle F \rangle = \langle \tilde{F} \rangle = 0. \end{aligned} \tag{3.2}$$

Let us expand around on a point on (3.2), as done in [1]. Then the remaining relevant terms of superpotential are given by

$$W_{dual}^{rel} = \Phi_0 (\delta\varphi \delta\tilde{\varphi} + m) + \delta Z \delta\varphi \tilde{q}_0 + \delta\tilde{Z} q_0 \delta\tilde{\varphi}$$

by following the fluctuations for the various fields in [11]. Note that there exist three kinds of terms, the vacuum $\langle q' \rangle$ multiplied by $\delta Y \delta\tilde{F}$, the vacuum $\langle \tilde{q}' \rangle$ multiplied by $\delta F \delta\tilde{Y}$, and the vacuum $\langle \Phi \rangle$ multiplied by $\delta Y \delta\tilde{Y}$. By redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [22], one gets that $m_{\Phi_0}^2$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

Let us recombine \tilde{N}'_c flavor D4-branes among the N'_f flavor D4-branes with those connecting NS5'-brane(coming from $NS5_{-\theta}$ -brane) and NS5-brane(coming from $NS5_{\theta'}$ -brane) and then push them in the $+v$ direction from the magnetic brane configuration in Figure 3. Then their mirrors will move $-v$ direction due to the presence of O6-plane. There are no color D4-branes connecting NS5'-brane and NS5-brane and there exist only $(N'_f - \tilde{N}'_c)$ flavor D4-branes connecting D6-branes and NS5'-brane(and their mirrors) that are misaligned to the \tilde{N}'_c flavor D4-branes.

The minimal energy supersymmetry breaking brane configuration is given by Figure 5. If we are detaching NS5-brane to the x^7 direction, then this brane configuration leads to the one discussed in [8] with opposite RR charge of O6-plane where the gauge group was $Sp(N_c) \times SU(\tilde{N}'_c)$ with fundamentals for each gauge group and bifundamentals. Of course, the brane configuration in Figure 5 with opposite O6-plane and without a middle NS5-brane is exactly the same as the brane configuration of [8].

Since we are dealing with five NS5-branes, the magnetic M5-brane configuration with equal mass for Q' and \tilde{Q}' and massless for Q and \tilde{Q} can be characterized by the following quintic equation for t as follows:

$$\begin{aligned} & t^5 + \left[v^{\tilde{N}'_c} \right] t^4 + \left[v^{N_c} \right] t^3 + \left[(-1)^{N_c} v^{N_c+N_f+2} (v+m')^{N'_f} \right] t^2 \\ & + \left[(-1)^{\tilde{N}'_c+N_f+N'_f} v^{\tilde{N}'_c+3N_f+6} (v+m')^{2N'_f} (v-m')^{N'_f} \right] t + \left[v^{10+5N_f} (v-m')^{2N'_f} (v+m')^{3N'_f} \right] \\ & = 0. \end{aligned}$$

Now the asymptotic regions of various NS5-branes can be determined by reading off the first two terms of the quintic curve above giving the $NS5'_L$ -brane asymptotic region, next two terms giving $NS5_L$ -brane asymptotic region, next two terms giving $NS5_M$ -brane asymptotic region, next two terms giving $NS5_R$ -brane asymptotic region, and final two terms giving $NS5'_R$ -brane asymptotic region. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

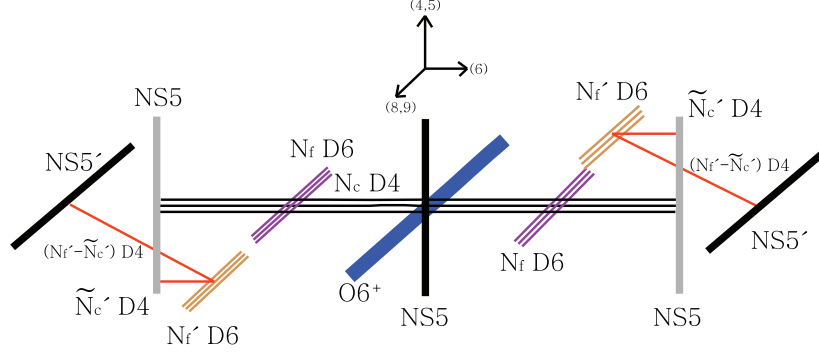


Figure 5: The nonsupersymmetric minimal energy brane configuration of $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c - N'_c)$ gauge group with fundamentals Q, \tilde{Q}, q' and \tilde{q}' for each gauge group, the bifundamentals Y and \tilde{Y} and a symmetric flavor S and a conjugate symmetric flavor \tilde{S} for $SU(N_c)$ and various gauge singlets. We consider the massless case of Q and \tilde{Q} . Compared with Figure 3, there is a misalignment of the N'_f flavor D4-branes. Some of them are connecting to NS5'-brane and the other to NS5-brane. In this figure, we put $\theta = \frac{\pi}{2}$ and $\theta' = 0$. The N_c D4-branes can move freely along the v -direction.

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{N_c - \tilde{N}'_c} + \dots & NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{N_f + N'_f + 2} + \dots & NS5_M \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{\tilde{N}_c - N_c + 2N_f + 2N'_f + 4} + \dots & NS5_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v &\rightarrow -m', & y &\sim w^{\tilde{N}'_c} + \dots & NS5'_L \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{-\tilde{N}'_c + 2N_f + 2N'_f + 4} + \dots & NS5'_R \text{ asymptotic region.} \end{aligned}$$

The two $NS5'_{L,R}$ -branes are moving in the $\mp v$ direction holding everything else fixed instead of moving D6-branes in the $\mp v$ direction. The corresponding mirrors of D4-branes are moved appropriately. Since the extra terms in the harmonic function contribute to the $g(s)$ as a multiplication factor, the statement that y is not equal to zero even if $v = 0$, implies that there exists the instability from a new M5-brane mode at some point from the transition of SQCD-like theory description to M-theory description.

4 Discussions

So far, the intersecting brane configurations of type IIA string theory are summarized by two figures Figure 4 and Figure 5 corresponding to the meta-stable nonsupersymmetric vacua of an $\mathcal{N} = 1$ supersymmetric electric gauge theory with the gauge group $SU(N_c) \times SU(N'_c)$ with fundamentals for each gauge group, the bifundamentals and a symmetric flavor and a conjugate symmetric flavor for $SU(N_c)$. This is done by applying Seiberg dual to each gauge group independently and obtaining two $\mathcal{N} = 1$ supersymmetric dual magnetic gauge theories with dual matters including the gauge singlets.

One can also generalize to the same $SU(N_c) \times SU(N'_c)$ gauge theory with fundamentals for each gauge group, the bifundamentals and an antisymmetric flavor and a conjugate symmetric flavor for $SU(N_c)$. Let us describe how the intersecting brane configurations arise here.

4.1 Electric theory with $SU(N_c) \times SU(N'_c)$ gauge group

The gauge group is $SU(N_c) \times SU(N'_c)$ which is the same as before but the matter contents are different and are given as follows:

- N_f -chiral multiplets Q are in the representation $(\mathbf{N}_c, \mathbf{1})$, and N_f -chiral multiplets \tilde{Q} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1})$, under the gauge group
- Eight-chiral multiplets \hat{Q} are in the representation $(\mathbf{N}_c, \mathbf{1})$ under the gauge group
- N'_f -chiral multiplets Q' are in the representation $(\mathbf{1}, \mathbf{N}'_c)$, and N'_f -chiral multiplets \tilde{Q}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}}'_c)$, under the gauge group
- The flavor-singlet field X is in the bifundamental representation $(\mathbf{N}_c, \overline{\mathbf{N}}'_c)$, and its conjugate field \tilde{X} is in the bifundamental representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c)$, under the gauge group
- The flavor-singlet field A , which is in an antisymmetric tensor representation under the $SU(N_c)$, is in the representation $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c - 1), \mathbf{1})$, and its conjugate field \tilde{S} is in the representation $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + 1), \mathbf{1})$, under the gauge group

If there are no antisymmetric, conjugate symmetric tensors and eight fundamentals, this theory is described by the work of [6, 4, 5] from field theory analysis or corresponding brane dynamics. Ignoring the presence of the fields Q', \tilde{Q}', X and \tilde{X} , then this theory will reduce to a single gauge group $SU(N_c)$ with an antisymmetric flavor, conjugate symmetric flavor and fundamental flavors A, \tilde{S}, Q and \tilde{Q} studied in [6, 14, 15, 16, 10].

The coefficient of the beta function of the first gauge group is given by

$$b_{SU(N_c)} = 3N_c - (N_f + 4) - N'_c - \frac{1}{2}(N_c + 2) - \frac{1}{2}(N_c - 2)$$

by realizing the index of the antisymmetric and symmetric representations of $SU(N_c)$ gauge

group and the coefficient of the beta function of the second gauge group is given by

$$b_{SU(N'_c)} = 3N'_c - N'_f - N_c.$$

These values will change when we go to the magnetic theory. The classical superpotential is given by

$$\begin{aligned} W = & \mu A_d^2 + A_d A \tilde{S} + \lambda Q A_d \tilde{Q} + \mu' A_d'^2 + \lambda' Q' A_d' \tilde{Q}' + X A_d \tilde{X} + \tilde{X} A_d' X \\ & + \hat{Q} \tilde{S} \hat{Q} + m Q \tilde{Q} + m' Q' \tilde{Q}', \end{aligned} \quad (4.1)$$

where the coefficient functions are given by

$$\mu \equiv \tan\left(\frac{\pi}{2} - \theta\right), \quad \mu' \equiv \tan(\theta' - \theta), \quad \lambda \equiv \sin(\theta - \omega), \quad \lambda' \equiv \sin(\theta' - \theta - \omega').$$

Here the adjoint field for $SU(N_c)$ gauge group is denoted by A_d while the adjoint field for $SU(N'_c)$ gauge group is denoted by A_d' . The second term in (4.1) arises from the presence of antisymmetric flavor A and a conjugate symmetric flavor \tilde{S} . Except this, the eighth term and the last mass terms, the above superpotential becomes the one described in [15, 4]. Setting the fields $Q', \tilde{Q}', X, \tilde{X}$ and A_d' to zero, the superpotential becomes the one discussed in [14, 15, 16, 10]. After integrating out the adjoint fields A_d and A_d' , this superpotential at the particular rotation angles $\theta = 0$ and $\theta' = \frac{\pi}{2}$ will reduce to the last two mass-deformed terms since the coefficient functions $\frac{1}{\mu}$ and $\frac{1}{\mu'}$ vanish at this particular rotation angles. For the nonsupersymmetric brane configuration, we will consider this particular brane configuration with the constraint $\theta = 0$ and $\theta' = \frac{\pi}{2}$ all the time.

The type IIA brane configuration for this gauge theory can be constructed similarly and can be drawn as Figure 1 except that at the origin of (x^6, v, w) coordinates, there exist NS5'-brane, $O6^+$ -plane, $O6^-$ -plane and eight half D6-branes, instead of having NS5-brane and $O6^+$ -plane. One can denote this as $NS5'/O6/D6$ -branes, as described in [10]. If we are detaching $NS5_{\pm\theta'}$ -branes, $D6_{\pm\omega'}$ -branes and N'_c D4-branes (and its mirrors), this brane configuration will reduce to the one described in [14, 15, 16, 10]. If we are detaching a middle NS5-brane to the x^7 direction, then this will lead to the brane configuration [17, 8] with the gauge group $Sp(N_c) \times SU(N'_c)$ or $SO(N_c) \times SU(N'_c)$ with fundamentals for each gauge group and bifundamentals, depending on the RR charge of $O6$ -plane. If we are detaching all the branes living on the negative x^6 region, two $O6$ -planes and eight D6-branes, then this will become the brane configuration of [4], as we mentioned before.

4.2 Magnetic theory with $SU(\tilde{N}_c) \times SU(N'_c)$ gauge group

From the magnetic and electric brane configurations we did not present here, the linking number counting, as done in [10], implies that the number of dual color \tilde{N}_c is given by

$$\tilde{N}_c = 2(N_f + N'_c) - N_c + 4. \quad (4.2)$$

Compared with the electric theory, the magnetic brane configuration has different features where there are extra N_f D6-branes and newly created N_f flavor D4-branes for the gauge group $SU(N'_c)$.

The dual magnetic gauge group is $SU(\tilde{N}_c) \times SU(N'_c)$ and the matter contents are as follows:

- N_f -chiral multiplets q are in the representation $(\tilde{\mathbf{N}}_c, 1)$, N_f -chiral multiplets \tilde{q} are in the representation $(\tilde{\mathbf{N}}_c, 1)$, under the gauge group
- Eight-chiral multiplets \hat{q} are in the representation $(\tilde{\mathbf{N}}_c, 1)$ under the gauge group
- N'_f -chiral multiplets Q' are in the representation $(\mathbf{1}, \mathbf{N}'_c)$, and N'_f -chiral multiplets \tilde{Q}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}'_c})$, under the gauge group
- The flavor-singlet field Y is in the bifundamental representation $(\tilde{\mathbf{N}}_c, \overline{\mathbf{N}'_c})$, and its complex conjugate field \tilde{Y} is in the bifundamental representation $(\tilde{\mathbf{N}}_c, \mathbf{N}'_c)$, under the gauge group
- The flavor-singlet field a , which is in an antisymmetric tensor representation under the $SU(\tilde{N}_c)$, is in the representation $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c - \mathbf{1}), \mathbf{1})$, and the conjugate symmetric field \tilde{s} is in the representation $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c + \mathbf{1}), \mathbf{1})$, under the gauge group

There are also $(N_f + N'_c)^2$ gauge-singlets in the first dual gauge group factor as follows:

- N_f -fields F' are in the representation $(\mathbf{1}, \mathbf{N}'_c)$, and its complex conjugate N_f -fields \tilde{F}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}'_c})$, under the gauge group
 - N_f^2 -fields M' are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
 - The $N_c'^2$ -fields Φ' is in the representation $(\mathbf{1}, \mathbf{N}_c'^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the gauge group
- Moreover, there are additional $N_f(2N_f + 1)$ gauge singlets
- N_f^2 -fields N' are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
 - N_f -fields \tilde{M} are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
 - $\frac{1}{2}N_f(N_f + 1)$ -fields P' are in the representation $(\mathbf{1}, \mathbf{1})$, and its conjugate $\frac{1}{2}N_f(N_f + 1)$ -fields \tilde{P}' are in the representation $(\mathbf{1}, \mathbf{1})$, under the gauge group

These are represented by $N' \equiv Q\tilde{S}A\tilde{Q}$, $\tilde{M} \equiv \hat{Q}\tilde{Q}$, $P' \equiv Q\tilde{S}Q$ and $\tilde{P}' \equiv \tilde{Q}A\tilde{Q}$ in terms of fields in electric theory, as observed in [6, 10]. Although these gauge-singlets N' , P' , and \tilde{P}' appear in the dual magnetic superpotential for the general rotation angles θ and θ' , the case $\theta = 0$ we are considering does not contain these gauge singlets, as found in [10].

The type IIA magnetic brane configuration can be described as in Figure 2 similarly, by replacing NS5-brane and $O6^+$ -plane by the combination of $NS5'/O6/D6$ -branes around at the origin, as mentioned before.

The coefficient of the beta function of the first dual gauge group factor is given by

$$b_{SU(\tilde{N}_c)}^{mag} = 3\tilde{N}_c - (N_f + 4) - N'_c - \frac{1}{2}(\tilde{N}_c + 2) - \frac{1}{2}(\tilde{N}_c - 2)$$

and the coefficient of the beta function of the second gauge group factor is given by

$$b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - \tilde{N}_c - N_f - N'_c.$$

It is clear that the $SU(N'_c)$ fields in the magnetic theory are different from those of the electric theory and this will lead to the various interaction terms in the dual magnetic superpotential⁵.

The dual magnetic superpotential⁶ for massless fundamental flavors Q' and \tilde{Q}' and massive fundamental flavors Q and \tilde{Q} is given by

$$W_{dual} = \left(M' q \tilde{s} a \tilde{q} + m M' + \hat{q} \tilde{s} \hat{q} + \tilde{M} \hat{q} \tilde{q} \right) + \tilde{Y} \tilde{F}' q + Y \tilde{q} F' + \Phi' Y \tilde{Y} + \left(\Phi'^2 + \dots \right) \quad (4.3)$$

where the mesons are given in terms of fields in the electric theory

$$M' \equiv Q \tilde{Q}, \quad \tilde{M} \equiv \hat{Q} \tilde{Q}, \quad F' \equiv \tilde{X} Q, \quad \tilde{F}' \equiv X \tilde{Q}, \quad \Phi' \equiv X \tilde{X}.$$

⁵More explicitly the conditions $b_{SU(\tilde{N}_c)}^{mag} < 0$ and $b_{SU(N_c)} > 0$ imply that $N_f + N'_c < \frac{2}{3}N_c - \frac{4}{3}$. Also the number of dual colors \tilde{N}_c defined as (4.2) should be positive. Then the range for the N_f in the first gauge group can be written as $\frac{1}{2}N_c - 2 < N_f + N'_c < \frac{2}{3}N_c - \frac{4}{3}$. Since $b_{SU(N'_c)} - b_{SU(N'_c)}^{mag} = 3(N'_c + N_f) - 2N_c + 4 < 0$, if we require that $b_{SU(N'_c)}^{mag} < 0$ which is equivalent to $N_c - 3N_f - 4 < N'_f$, then the electric description of $SU(N'_c)$ is IR free because $b_{SU(N'_c)} < 0$. At high energy, $SU(N'_c)$ theory is strongly coupled while $SU(N_c)$ theory is UV free. At the scale Λ_1 , the $SU(N_c)$ theory is strongly coupled and the Seiberg duality occurs. All the running couplings are changed by this duality and all the coefficients of beta functions, $b_{SU(\tilde{N}_c)}^{mag}$ and $b_{SU(N'_c)}^{mag}$ become negative. Then at energy scale lower than Λ_1 , the theory is weakly coupled. When $b_{SU(N'_c)}^{mag} < b_{SU(\tilde{N}_c)}^{mag} < 0$, the one loop computation is reliable with $\Lambda_1 \ll \Lambda_2$. When $b_{SU(\tilde{N}_c)}^{mag} < b_{SU(N'_c)}^{mag} < 0$, the requirement that $SU(N'_c)^{mag}$ theory is less coupled than the $SU(\tilde{N}_c)^{mag}$ at the supersymmetry breaking scale μ provides a stronger constraint on Λ_2 . Then under the constraint, $\Lambda_2 \gg \left(\frac{\Lambda_1}{\mu}\right)^b \Lambda_1$ where b is defined as in footnote 1, one can ignore the contribution from the gauge coupling of $SU(N'_c)^{mag}$ at the supersymmetry breaking scale and one relies on the one loop computation.

⁶We integrate out the bifundamentals Y and \tilde{Y} and \hat{q} in such a way that the gauge group $SU(\tilde{N}_c)$ is not broken by the fields Y and \tilde{Y} and \hat{q} , as in meta-stable state, so $\langle Y \rangle = 0 = \langle \tilde{Y} \rangle = \langle \hat{q} \rangle$ in (4.4). For nonzero vacuum expectation values for M' , this superpotential gives the $SU(\tilde{N}_c)$ “flavors” $q\tilde{s}$ and $a\tilde{q}$, the mass $\langle M' \rangle$. Below the energy scale $\langle M' \rangle$, one can integrate out these massive flavors using the equations of motion $\langle q\tilde{s} \rangle = 0 = \langle a\tilde{q} \rangle$. Then the low energy theory has a superpotential term which is proportional to $\left(\tilde{\Lambda}^{2\tilde{N}_c - N_f - N'_c - 4} \det M'\right)^{\frac{1}{\tilde{N}_c}}$ plus mM' . There is no conserved $U(1)_R$ symmetry because it is anomalous under the gauged $SU(\tilde{N}_c)$ in the sense that the determinant term above breaks it explicitly. Therefore, the $U(1)_R$ symmetry returns an “approximate” accidental symmetry of the IR theory.

The last piece $(\Phi'^2 + \dots)$ in W_{dual} is coming from the superpotential (4.1) in an electric theory and contains also $N'q\tilde{q} + P'q\tilde{s}q + \tilde{P}'\tilde{q}a\tilde{q}$ for the general rotation angles θ, θ', ω and ω' . When $\theta = 0$ and $\theta' = \frac{\pi}{2}$, this will vanish and the superpotential consists of the first seven terms in (4.3) which will play an important role when we discuss the nonsupersymmetric brane configuration. Also the above mesons can be interpreted as strings connecting various D-branes, as we did in previous section.

The dual quarks q and \tilde{q} are fundamental $(\tilde{\mathbf{N}}_c, \mathbf{1})$ and antifundamental $(\overline{\tilde{\mathbf{N}}_c}, \mathbf{1})$ for the gauge group indices and antifundamentals for the flavor indices. The flavor-singlet fields a and \tilde{s} are antisymmetric $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c - \mathbf{1}), \mathbf{1})$ and conjugate symmetric tensor $(\frac{1}{2}\tilde{\mathbf{N}}_c(\tilde{\mathbf{N}}_c + \mathbf{1}), \mathbf{1})$ for the gauge group indices respectively. The quantity $q\tilde{s}a\tilde{q}$ has a rank \tilde{N}_c and the mass matrix m has a rank N_f . Then the F-term equation for M' cannot be satisfied if the rank N_f exceeds \tilde{N}_c and the supersymmetry is broken.

The classical moduli space of vacua can be obtained from F-term equations and one gets

$$\begin{aligned} q\tilde{s}a\tilde{q} + m &= 0, & \tilde{s}a\tilde{q}M' + \tilde{Y}\tilde{F} &= 0, \\ a\tilde{q}M'q + \hat{q}\hat{q} &= 0, & \tilde{q}M'q\tilde{s} &= 0, \\ M'q\tilde{s}a + \tilde{M}\hat{q} + F'Y &= 0, & \tilde{F}'q + \Phi'Y &= 0, \\ q\tilde{Y} &= 0, & \tilde{q}F' + \tilde{Y}\Phi' &= 0, \\ Y\tilde{q} &= 0, & Y\tilde{Y} + \Phi' &= 0, \\ \tilde{s}\hat{q} + \tilde{q}\tilde{M} &= 0, & \hat{q}\tilde{q} &= 0. \end{aligned}$$

Some of F-term equations are satisfied if one takes the zero vacuum expectation values for the fields $Y, \tilde{Y}, F', \tilde{F}', \hat{q}$ and \tilde{M} . Then, it is easy to see that

$$a\tilde{q}M' = 0 = M'q\tilde{s}, \quad q\tilde{s}a\tilde{q} + m = 0.$$

Then the solutions can be written as

$$\begin{aligned} \langle q\tilde{s} \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \quad \langle a\tilde{q} \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \quad \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{pmatrix} \\ \langle Y \rangle &= \langle \tilde{Y} \rangle = \langle F' \rangle = \langle \tilde{F}' \rangle = \langle \hat{q} \rangle = \langle \tilde{M} \rangle = 0. \end{aligned} \quad (4.4)$$

Let us expand around on a point on (4.4), as done in [1]. Then the remaining relevant terms of superpotential are given by

$$W_{dual}^{rel} = \Phi_0 \left(\delta\hat{\varphi} \delta\hat{\tilde{\varphi}} + m \right) + \delta Z \delta\hat{\varphi} a_0 \tilde{q}_0 + \delta\tilde{Z} q_0 \tilde{s}_0 \delta\hat{\tilde{\varphi}}$$

by following the fluctuations for the various fields in [10]. Note that there exist five kinds of terms, the vacuum $\langle q \rangle$ multiplied by $\delta\tilde{Y}\delta\tilde{F}'$, the vacuum $\langle \tilde{q} \rangle$ multiplied by $\delta F'\delta Y$, the

vacuum $\langle \Phi' \rangle$ multiplied by $\delta Y \delta \tilde{Y}$, the vacuum $\langle \tilde{s} \rangle$ multiplied by $\delta \hat{q} \delta \hat{q}$, and the vacuum \tilde{q} multiplied by $\delta \tilde{M} \delta \hat{q}$. By redefining these as before, they do not enter the contributions for the one loop result, up to quadratic order. As done in [11], the defining function $\mathcal{F}(v^2)$ can be computed and using the equation (2.14) of [22] of $m_{\Phi_0}^2$ and $\mathcal{F}(v^2)$, one gets that $m_{\Phi_0}^2$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

Then the minimal energy supersymmetry breaking brane configuration is given by Figure 6. If we are detaching N'_c D4-branes, N'_f D6-branes and NS5'-brane(coming from $NS5_{\theta'}$ -brane)(and its mirrors), then this brane configuration leads to the one described in [10].

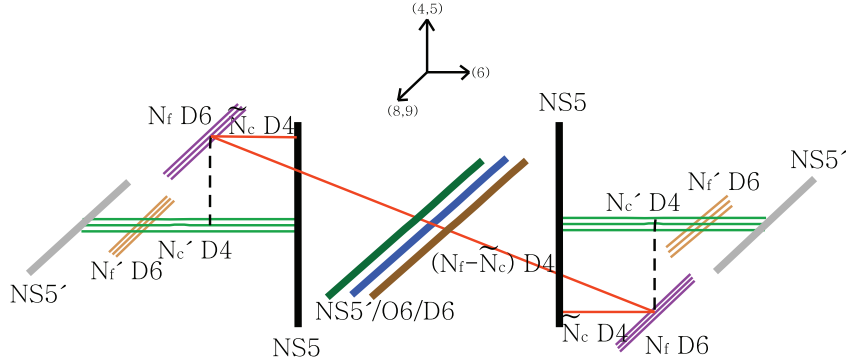


Figure 6: The nonsupersymmetric minimal energy brane configuration of $SU(\tilde{N}_c = 2N_f + 2N'_c - N_c + 4) \times SU(N'_c)$ gauge group with fundamentals $q, \tilde{q}, \hat{q}, Q'$ and \tilde{Q}' for each gauge group, the bifundamentals Y and \tilde{Y} and an antisymmetric flavor a and a conjugate symmetric flavor \tilde{s} for $SU(\tilde{N}_c)$ and various gauge singlets. We consider the massless case of Q' and \tilde{Q}' . From the modified Figure 2, there is a misalignment of N_f flavor D4-branes. Some of them are connecting to NS5'-brane and the other to NS5-brane. In this figure, we put $\theta = 0$ and $\theta' = \frac{\pi}{2}$.

Now the asymptotic regions of various NS5-branes, at nonzero string coupling constant, can be determined by reading off the first two terms of the quintic curve given in subsection 3.1 giving the $NS5'_L$ -brane asymptotic region, next two terms giving $NS5_L$ -brane asymptotic region, next two terms giving $NS5'_M$ -brane asymptotic region, next two terms giving $NS5_R$ -brane asymptotic region, and final two terms giving $NS5'_R$ -brane asymptotic region. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{\tilde{N}_c - N'_c + N_f + N'_f} + \dots && NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{N'_c - \tilde{N}_c + N_f + N'_f + 4} + \dots && NS5_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v &\rightarrow 0, \quad y \sim w^{N'_c} + \dots \quad NS5'_L \text{ asymptotic region,} \\ v &\rightarrow \pm m, \quad y \sim w^{N_f+N'_f+2} + \dots \quad NS5'_M \text{ asymptotic region,} \\ v &\rightarrow 0, \quad y \sim w^{2N_f+2N'_f-N'_c+4} + \dots \quad NS5'_R \text{ asymptotic region.} \end{aligned}$$

The harmonic function in the Tau-NUT space, sourced by $2N_f$ D6-branes, O6-plane and $2N'_f$ D6-branes, can be determined once we fix the x^6 position for these branes and write the charges. Then the first order differential equation for the $g(s)$ where the absolute value of $g(s)$ is equal to the absolute value of w can be solved exactly with the appropriate boundary conditions on $NS5'_M$ asymptotic region from above classification 2. Since the extra terms in the harmonic function contribute to the $g(s)$ as a multiplication factor, the contradiction with the correct statement that y should vanish only if $v = 0$, implies that there exists the instability from a new M5-brane mode.

4.3 Magnetic theory with $SU(N_c) \times SU(\tilde{N}'_c)$ gauge group

Now we continue to analyze for other magnetic theory. From the magnetic and electric brane configurations, the linking number counting implies that the number of dual color \tilde{N}'_c is given by, as we did in (2.4),

$$\tilde{N}'_c = N'_f + N_c - N'_c. \quad (4.5)$$

The dual magnetic gauge group is given by $SU(N_c) \times SU(\tilde{N}'_c)$ and the matter contents are as follows:

- N_f -chiral multiplets Q are in the representation $(\mathbf{N}_c, \mathbf{1})$, and N_f -chiral multiplets \tilde{Q} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1})$, under the gauge group
- Eight-chiral multiplets \hat{Q} are in the representation $(\mathbf{N}_c, \mathbf{1})$ under the gauge group
- N'_f -chiral multiplets q' are in the representation $(\mathbf{1}, \tilde{\mathbf{N}}'_c)$, N'_f -chiral multiplets \tilde{q}' are in the representation $(\mathbf{1}, \overline{\tilde{\mathbf{N}}}'_c)$, under the gauge group
- The flavor singlet field Y is in the bifundamental representation $(\mathbf{N}_c, \overline{\tilde{\mathbf{N}}}'_c)$, and its complex conjugate field \tilde{Y} is in the bifundamental representation $(\overline{\mathbf{N}}_c, \tilde{\mathbf{N}}'_c)$, under the gauge group
- The flavor singlet field A , which is in an antisymmetric tensor representation under the $SU(N_c)$, is in the representation $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c - \mathbf{1}), \mathbf{1})$, and its conjugate field \tilde{S} is in the representation $(\frac{1}{2}\overline{\mathbf{N}}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1})$, under the gauge group

There are $(N'_f + N_c)^2$ gauge singlets in the second dual gauge group factor as follows:

- N'_f -fields F are in the representation $(\mathbf{N}_c, \mathbf{1})$, and its complex conjugate N'_f -fields \tilde{F} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1})$, under the gauge group

- $N_f'^2$ -fields M are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group
- The N_c^2 -fields Φ is in the representation $(\mathbf{N}_c^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the gauge group

The type IIA magnetic brane configuration can be described as in Figure 3, by replacing NS5-brane and $O6^+$ -plane by the combination of $NS5'/O6/D6$ -branes around at the origin, as mentioned before. We'll not present those here.

The coefficient of the beta function of the first gauge group factor is given by

$$b_{SU(N_c)}^{mag} = 3N_c - (N_f + 4) - \tilde{N}_c' - N_f' - N_c - \frac{1}{2}(N_c + 2) - \frac{1}{2}(N_c - 2)$$

as before and the coefficient of the beta function of the second gauge group factor is given by

$$b_{SU(\tilde{N}_c')}^{mag} = 3\tilde{N}_c' - N_f' - N_c.$$

Since $b_{SU(N_c)} - b_{SU(N_c)}^{mag} > 0$, the $SU(N_c)$ theory is more asymptotically free ⁷ than the $SU(N_c)^{mag}$ theory [5].

The dual magnetic superpotential ⁸ for massless fundamental flavors Q and \tilde{Q} (i.e., $m = 0$) and massive fundamental flavors Q' and \tilde{Q}' is

$$W'_{dual} = \left(M q' \tilde{q}' + m' M \right) + Y \tilde{F} q' + \tilde{Y} q' F + \Phi Y \tilde{Y} + (\Phi^2 + \dots) \quad (4.6)$$

where the mesons are given by

$$M \equiv Q' \tilde{Q}', \quad F \equiv X Q', \quad \tilde{F} \equiv \tilde{X} \tilde{Q}', \quad \Phi \equiv X \tilde{X}.$$

Here the last piece $(\Phi^2 + \dots)$ in W_{dual} is coming from the superpotential (4.1) in an electric theory for the general rotation angles θ, θ', ω and ω' . When $\theta = 0$ and $\theta' = \frac{\pi}{2}$, this will vanish

⁷More explicitly, the conditions $b_{SU(\tilde{N}_c')}^{mag} < 0$ and $b_{SU(N_c)} > 0$ imply that $N_f' + N_c < \frac{3}{2}N_c'$. Also the number of dual colors \tilde{N}_c' defined as (4.5) should be positive. Then the range for the N_f' in the second gauge group can be written as $N_c' < N_f' + N_c < \frac{3}{2}N_c'$. The condition $b_{SU(N_c)}^{mag} < 0$ implies $N_c' - 2N_f' - 4 < N_f$. The $b_{SU(N_c)}$ can be IR free or UV free in the electric description. One can easily analyze four different possibilities as in footnote 3. Two cases for the positivity or negativity of the difference between $b_{SU(\tilde{N}_c')}^{mag}$ and $b_{SU(N_c)}^{mag}$ and two cases for the UV free or IR free for $SU(N_c)$ theory in an electric description.

⁸ For nonzero vacuum expectation values for M , this superpotential gives the $SU(\tilde{N}_c')$ fundamental flavors q' and \tilde{q}' , the mass $\langle M \rangle$. Below the energy scale $\langle M \rangle$, one can integrate out these massive flavors using the equations of motion $\langle q' \rangle = 0 = \langle \tilde{q}' \rangle$. Then the low energy theory has a superpotential term which is proportional to $\left(\tilde{\Lambda}^{3\tilde{N}_c' - N_f' - N_c} \det M \right)^{\frac{1}{N_c'}}$ plus $m' M$. There is no conserved $U(1)_R$ symmetry because it is anomalous under the gauged $SU(\tilde{N}_c')$ in the sense that the determinant term above breaks it explicitly. Remember that the R charges for the fields are as follows: $R(Y) = R(q') = \frac{N_c'}{N_f' + N_c}$ and $R(M) = R(F) = R(\Phi) = 2 - \frac{2N_c'}{N_f' + N_c}$. Therefore, the $U(1)_R$ symmetry returns an “approximate” accidental symmetry of the IR theory.

and the superpotential consists of the first five terms in (4.6). Also the above mesons can be interpreted as strings connecting various D-branes, as before.

The dual quarks q' and \tilde{q}' are fundamental $(\mathbf{1}, \tilde{\mathbf{N}}'_c)$ and antifundamental $(\mathbf{1}, \overline{\tilde{\mathbf{N}}}'_c)$ for the gauge group indices and antifundamentals for the flavor indices. The quantity $q'\tilde{q}'$ has a rank \tilde{N}'_c and the mass matrix m' has a rank N'_f . Then the F-term equation for M cannot be satisfied if the rank N'_f exceeds \tilde{N}'_c and the supersymmetry is broken.

The classical moduli space of vacua can be obtained from F-term equations and one gets

$$\begin{aligned} q'\tilde{q}' + m' &= 0, & \tilde{q}'M + Y\tilde{F} &= 0, \\ Mq' + F\tilde{Y} &= 0, & \tilde{F}q' + \tilde{Y}\Phi &= 0, \\ q'Y &= 0, & \tilde{q}'F + \Phi Y &= 0, \\ \tilde{Y}\tilde{q}' &= 0, & Y\tilde{Y} &= 0. \end{aligned}$$

Other F-term equations are satisfied if one takes the zero vacuum expectation values for the fields Y, \tilde{Y}, F and \tilde{F} . Then, it is easy to see that

$$\tilde{q}'M = 0 = Mq', \quad q'\tilde{q}' + m' = 0.$$

Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix} \\ \langle Y \rangle &= \langle \tilde{Y} \rangle = \langle F \rangle = \langle \tilde{F} \rangle = 0. \end{aligned} \quad (4.7)$$

Let us expand around on a point on (4.7), as done in [1]. Then the remaining relevant terms of superpotential are given by

$$W_{dual}^{rel} = \Phi_0 (\delta\varphi \delta\tilde{\varphi} + m) + \delta Z \delta\varphi \tilde{q}_0 + \delta\tilde{Z} q_0 \delta\tilde{\varphi}$$

by following the fluctuations for the various fields in [11]. Note that there exist three kinds of terms, the vacuum $\langle q' \rangle$ multiplied by $\delta Y \delta\tilde{F}$, the vacuum $\langle \tilde{q}' \rangle$ multiplied by $\delta F \delta\tilde{Y}$, and the vacuum $\langle \Phi \rangle$ multiplied by $\delta Y \delta\tilde{Y}$. By redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [22], one gets that $m_{\Phi_0}^2$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

Then the minimal energy supersymmetry breaking brane configuration is given by Figure 7. If we are moving NS5'-brane to $\pm x^7$ direction, then the brane configuration will lead to the Figure 6 of [8] where the gauge group is $SO(N_c) \times SU(\tilde{N}'_c)$ or $Sp(N_c) \times SU(\tilde{N}'_c)$, depending on the movement of NS5'-brane to $+x^7$ direction or $-x^7$ direction, with fundamentals for each gauge group and bifundamentals.

that there exists the instability from a new M5-brane mode at some point from the transition of SQCD-like theory description to M-theory description.

It is natural to ask if we can generalize the procedure for the product gauge group of two gauge groups, which will contain three NS5-branes, four NS5-branes or five NS5-branes, to configurations with more than the product gauge group of three gauge groups. When there exist three NS5-branes with two gauge groups, one can add an extra NS5-brane to the left of left NS5-brane or to the right of right NS5-brane for the triple gauge groups. This extra NS5-brane will be perpendicular to its closest NS5-brane. One can also add an orientifold 4-plane in this brane configuration. When there are four NS5-branes with O6-plane with two gauge groups, we add two outer NS5-branes in a \mathbf{Z}_2 symmetric way and each outer NS5-brane will be perpendicular to its closest NS5-brane for the triple gauge groups. When there are five NS5-branes with two gauge groups, one adds two outer NS5-branes in a \mathbf{Z}_2 symmetric way and each outer NS5-brane will be perpendicular to its closest NS5-brane for the triple gauge groups. For each case, one needs to understand how the magnetic superpotential arises from its electric theory and to analyze the corresponding F-term equations.

There exist different directions concerning on the meta-stable vacua in different contexts. Some of the relevant works are present in recent works [28]-[38] where some of them use anti D-branes and some of them are described in the type IIB theory. It would be interesting to find out how similarities and differences between type IIA and IIB theories arise.

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